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P. Di Vecchia : A NOTE ON DOUBLE BREMSSTRAHLUNG.

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A Note on Double Bremsstrahlung.

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The first attempts at monitoring the common volume of the colliding beams in an electron-positron storage ring consisted in trying to observe the coincidences caused by two-photon annihilation [1]. The experiment, which used lead glass Čerenkov counters to detect photons of more than about 70 MeV energy produced in the collision region by electrons and positrons of about 200 MeV each, revealed the existence of a continuous spectrum for the sum of the energies of the two photons. This continuum was attributed to the process of double bremsstrahlung, viz.:

$$e^+ + e^- \to e^+ + e^- + 2\gamma \, . \tag{1}$$

Since process (1) can be conducted in such a way that the four-momentum transfer between the colliding particles could be made arbitrarily small provided that the incident energy is sufficiently large, it was concluded that (1) would dominate over two-quanta annihilation at very high energies by a factor of the order $\alpha^2(E/m)^2$. This as yet unproven property of the process (1) seemed to place it on the list of possible monitoring processes defined as collision processes between electrons and positrons in which the momentum transfer could be considered sufficiently small to justify the assumption, that their description could be entrusted to quantum electrodynamics with unit form-factors and unmodified propagators. The processes on the list were: backward-forward two-quanta annihilation, small angle e^+e^- scattering, single e^+e^- bremsstrahlung. The following points seemed to be in favour of adopting (1) as a monitoring process:

1) Its cross-section is large compared to the cross-section for two-quanta annihilation.

2) The angular dependence of the emitted photons is less critical than in the case of e^+e^- scattering; it is finite in the forward direction, whereas the cross-section for scattering is not.

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(1) C. BERNARDINI, G. F. CORAZZA, G. DI GIUGNO, J. HAISSINSKI, P. MARIN, R. QUERZOLI and B. TOUSCHEK: *Nuovo Cimento*, **34**, 1473 (1964).

3) Being observed by a coincidence experiment, its advantage over the process of single bremsstrahlung seemed to be that some of the background could be successfully eliminated by using a sufficiently high time resolution of the coincidences.

BAYER and GALITSKY [2] have given an approximate formula valid for small energies of the emitted photons: ω_1, $\omega_2 \ll E$. Their result is

$$\mathrm{d}\sigma^{(2)} = \frac{8r_0^2\alpha^2}{\pi}\frac{\mathrm{d}\omega_1}{\omega_1}\frac{\mathrm{d}\omega_2}{\omega_2}\left[\frac{5}{4}+\frac{7}{8}\,\xi(3)\right] \tag{2}$$

where $\xi(3) = \sum_1^\infty n^{-3}$ and $r_0 = 2.8\cdot 10^{-13}\,\mathrm{cm}^2$ is the classical electron radius. This result has been checked by the author, who by the use of the Bloch-Nordsieck [3] method obtained:

$$\mathrm{d}\sigma^{(2)} = \frac{8r_0^2\alpha^2}{\pi}\frac{\mathrm{d}\omega_1}{\omega_1}\frac{\mathrm{d}\omega_2}{\omega_2}\left[2+\frac{7}{8}\,\xi(3)\right]. \tag{3}$$

The Bayer-Galitsky cross-section is about 25% less than the one given in (3). It has not been possible to trace this discrepancy between (2) and (3), the latter formula is however borne out by the more detailed calculation presented in the following and by cross checks with the unpublished work of GRECO on the same subject. The low-energy limit of double bremsstrahlung is not easily accessible to experiment, since we must assume that the background increases rapidly with decreasing energy of the two photons.

In order to obtain information about the high-energy behaviour of the process, the special case in which one photon follows exactly the direction of the electron, the other the direction of the positrons has been treated in detail. Only the leading graphs have been considered, so that the following result is valid $0(I/\gamma)$, where $\gamma = E/m$ (E could be the energy of the final electrons, so that the results presented here will be wrong in the immediate vicinity of the « head » of the spectrum).

From what was said earlier about the angular distribution, the result should be significant for a detection device of about 1 cm² effective area placed at a distance of about 10 m from the collision region of « ADONE » (*). (Such a device could be a spark chamber, viewing a larger angular range. The peak cross-section could be used as a calibration of this device.)

The cross-section for the backward-forward process is:

$$\mathrm{d}\sigma^{(2)*} = \frac{\alpha^2 r_0^2}{\pi^3}\frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\left[\frac{4}{3}(1-x_1)(1-x_2)+(1-x_1)x_2^2+(1-x_2)x_1^2+ x_1^2x_2^2\right]\gamma^4\,\mathrm{d}\Omega_1\,\mathrm{d}\Omega_2 \tag{4}$$

where $x_{1,2} = \omega_{1,2}/E$ and $\mathrm{d}\Omega_{1,2}$ are the elements of solid angle of the two photons,

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([2]) BAYER and R. R. GALITSKY: *Phys. Lett.*, **13**, 355 (1964).
([3]) F. BLOCH and A. NORDSIECK: *Phys. Rev.*, **52**, 54 (1937).
(*) But compare remark at the end of this paper.

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(4) is only valid for

$$d\Omega_{1,2} \ll \frac{\pi}{\gamma^2}. \tag{5}$$

The cross-section (4) has been checked by using the Bloch-Nordsieck method, which gives:

$$d\sigma^{(2)*}_{\text{B.N.}} = \frac{4\alpha^2 r_0^2}{3\pi^3} \frac{dx_1}{x_1} \frac{dx_2}{x_2} \gamma^4 \, d\Omega_1 \, d\Omega_2 \tag{6}$$

and it is seen that (6) and (4) agree in the limit $x_{1,2} \to 0$. A relief map of the function

$$\frac{d\sigma^{(2)*} \pi^3 x_1 x_2}{\alpha^2 r^2 \gamma^4 \, dx_1 \, dx_2 \, d\Omega_1 \, d\Omega_2}$$

is given in Fig. 1. A remarkable feature of both the results (3) and (4) is the absence of a logarithmic term of the form $\log 2\gamma$. This puts double bremsstrahlung at a disadvantage against the background of chance coincidences due to single bremsstrahlung created either in collision with the molecules of the residual gas or in genuine electron-positron collisions ([4]).

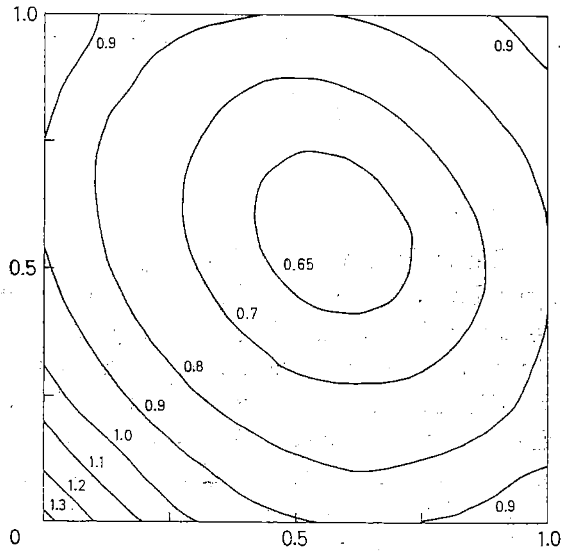

Fig. 1.

At present we are not in a position to decide whether this absence of the logarithmic term is due to the approximation (5). It is indeed quite possible that such a term might appear in the total cross-section for process (1), though the mechanism responsible for the appearance of a logarithmic term must be different from the one which acts in the case of single bremsstrahlung, where it is produced as a consequence of the fact that the probability of scattering by an angle θ in a Coulomb field is proportional to $d\theta/\theta^3$ and that because of gauge invariance the modulus square of the current matrix element is proportional to θ^2, so that the total probability of single radiative scattering diverges logarithmically at $\theta = 0$. In the case of process (1) there is no such divergence at $\theta = 0$, since this process is proportional to the fourth power of the current.

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([4]) S. TAZZARI: Unpublished paper presented at the *Congressino ADONE*, Frascati (February 1966).